K22U 3420

Reg. No. : MG22CPHR15

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I Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01MAT-PH : Mathematics for Physics – I

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions from among the questions **1** to **5**. **Each** question carries **one** mark.

1. Find the n^{th} derivative of $\sin(ax + b)$.
2. State generalized mean value theorem.
3. State Rouche's theorem.
4. Prove that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular.
5. Find polar equation for the circle $x^2 + (y - 3)^2 = 9$.

PART – B

Answer **any seven** questions from among the questions **6** to **16**. **Each** question carries **2** marks.

6. If $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.
7. If $y = (2 - 3x)^{10}$, find y_9.
8. If $y = e^{ax} \sin bx$, prove that $y_2 - 2ay_1 + (a^2 + b^2) y = 0$.
9. Verify Rolle's theorem for $f(x) = \frac{\sin x}{e^x}$ in $(0, \pi)$.

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10. Using Maclaurin's series, expand tan x upto the term containing x^5.

11. Find $\lim_{x \to 0} x^n \log x, n > 0$.

12. Evaluate $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

13. Determine the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

14. Using the Gauss-Jordan method, find the inverse of $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

15. If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal, find a, b, c and A^{-1}.

16. Find the spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

PART – C

Answer **any four** questions from among the questions **17** to **23**. **Each** question carries **three** marks.

17. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$.

18. If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.

19. Expand $\log(1 + \sin^2 x)$ in powers of x as far as term in x^6.

20. Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form and hence find its rank.

21. Solve the following system of equations by Cramer's rule
$3x + y + 2z = 3,\ 2x - 3y - z = -3,\ x + 2y + z = 4$.

22. Calculate $\frac{ds}{dx}$ for the curve $ay^2 = x^3$.

23. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the Folium $x^3 + y^3 = 3axy$

PART – D

Answer **any two** questions from among the questions **24** to **27**. **Each** question carries **five** marks.

24. State and prove Leibnitz's theorem for the n^{th} derivative of the product of two functions.

25. Evaluate

i) $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

ii) $\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x}$.

26. Find the value of λ for which the equations

$(\lambda - 1)\, x + (3\lambda + 1)\, y + 2\lambda z = 0$

$(\lambda - 1)\, x + (4\lambda - 2)\, y + (\lambda + 3)\, z = 0$

$2x + (3\lambda + 1)\, y + 3(\lambda - 1)\, z = 0$

are consistent, and find the ratios of $x : y : z$ when λ has the smallest of these values. What happens when λ has the greater of these values ?

27. Find the co-ordinates of the centre of curvature at any point of the parabola $y^2 = 4ax$.